Medium Damping Influences on the Resonant Frequency and Quality Factor of Piezoelectric Circular Microdiaphragm Sensors

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Abstract:
Medium damping influences on the resonant frequency and quality factor of piezoelectric circular microdiaphragm sensors (PCMS) are investigated theoretically and experimentally in this paper. The acoustic radiation and viscosity damping as the two main sources of energy dissipation in medium virtually added the mass of the diaphragm and therefore decrease the frequency and $Q$-factor of the diaphragm. The magnitude of medium damping inversely depends on the radius over thickness ratio. Increase of this ratio is the trend in fabrication of thin microdiaphragms by MEMS fabrication processes, which implies the higher influence of medium damping in dynamic behavior of microdiaphragms. The fabricated PCMS were tested in vacuum, air, and in ethanol. The $Q$-factor and the resonant frequency of the device increase by almost seven times and 4.7\% from air to 0.05 atm pressure, respectively. The $Q$-value drops from 111.195 in air to 23.908 in ethanol. Throughout the work, theoretical and experimental values were compared and fairly good correlation was observed.

1 Introduction:
Micro- or nanoelectromechanical resonant sensors are devices which their resonant frequency shifts as a function of a physical or chemical parameter [1-5]. They have been extensively used as highly sensitive sensors with different actuation mechanisms such as optical, electrostatic, electromagnetic, and piezoelectric for detecting gases, chemicals, or biological entities [6-9]. Modeling these sensors regardless of their shapes and applications as a simple one-dimensional damped harmonic oscillator will result the familiar equation of motion [10]

$$\ddot{x}(t) + \frac{a_0}{Q} \dot{x}(t) + a_0^2 x(t) = \frac{f(t)}{m_{\text{eff}}}$$

(1)

Here, $f(t)$ is the driving force, $m_{\text{eff}}$ is the effective mass, and $Q$ is the quality factor of the mode in question. For mass sensing, it is desirable to minimize $m_{\text{eff}}/Q$. In vacuum $m_{\text{eff}}$ is the mass of the resonator itself; however, for a resonator vibrates in a fluid, the mass of surrounding fluid will affects this mass which in turn reduces the resonant frequency of the sensor and degrades the mass sensitivity. The sensor $Q$-factor is defined as $Q=2\pi W_0/\Delta W$, where $W_0$ is the stored vibrational energy and $\Delta W$ is the total energy lost per cycle of vibration. The energy lost term can be written as $\Delta W = \sum_i \Delta W_i$, where $\Delta W_i$ represents the different dissipation mechanisms which contribute to the total energy lost. Overall, the total $Q$-factor ($Q_{\text{tot}}$) of a device can be written as [11]
The main energy dissipation mechanisms for a mechanical resonator can be identified as (a) medium loss which is the losses into the surrounding (fluid) medium due to acoustic radiation [12] or viscous drag [13] (b) clamping or support losses which is dissipation of energy through the support used to mount the resonator results from vibration of resonator [14, 15], and (c) bulk losses which is composed of variety of physical mechanisms, such as internal friction, thermoelastic dissipation (TED), phonon-phonon scattering, and motion of lattice defects [11].

These energy dissipation mechanisms are not equally contribute to the total energy lost of the system. Size and ambient pressure are the two main parameters, which clarify the contribution of these terms. Size reduction from macro to micro scale increases the resonator’s surface to volume ratio, hence signifies effect of surface forces, and dominates them over the body forces. Therefore, bulk losses are negligible in microscale regions compared to medium damping terms. For a resonator vibrates in air, the medium damping is heavily dependent to the ambient pressure [16]. Generally, the pressure range from atmosphere to high vacuum is divided to three different regions [17]. In atmospheric region air acts as a viscous fluid and the medium damping is dominant. In high vacuum region due to the elimination of surrounding air the bulk losses regain their significance and become the dominant dissipation mechanism. In between there is molecular region which the surface dissipation due to independent collision of non interacting air molecular with the moving surface of the resonator is the dominant.

Based on aforementioned notes, the dominant damping mechanism for piezoelectric circular microdiaphragm sensors (PCMS) working in normal atmosphere or in aqueous environment is the medium damping. Therefore, in this work we theoretically and experimentally investigate influences of this damping, which is composed of acoustic radiation and viscous terms, in the resonant frequency and quality factor of PCMS.

2 Theory

2.1 Plate vibrating in vacuum

Let us consider a thin circular plate made of linear elastic, homogeneous and isotropic material having radius \(a\), mass density \(p\), and thickness \(h\). The plate is clamped around its edges and is vibrating in vacuum. Moreover, the effects of shear deformation and rotary inertia are neglected. Lamb [12] in 1920, approximated the normal displacement profile of this plate by an assumed mode shape

\[
\frac{1}{Q_{\text{tot}}} = \frac{1}{Q_{\text{medium}}} + \frac{1}{Q_{\text{clamping}}} + \frac{1}{Q_{\text{bulk}}} + \frac{1}{Q_{\text{others}}} \tag{2}
\]

![Image](https://example.com/figure.png)

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\[
w(r,t) = X(t) \left(1 - \frac{r^2}{a^2}\right)^2
\]

Where \(w(r,t)\) is the transverse displacement, and \(X(t)\) is a function of time. It is worth noting that the exact mode shape of the plate vibrating in vacuum is a combination of Bessel functions as shown in equation (4) [18]; however, it was previously proved that the assumed mode shape is an adequate approximation [19].

\[
w(r,\theta,t) = \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} \left[A_{mn} J_m \left(\lambda_{mn} r / a\right) + B_{mn} I_m \left(\lambda_{mn} r / a\right)\right] \cos m(\theta - \varphi_n) e^{i\omega t} \tag{4}
\]
The maximal kinetic and potential energies of the diaphragm vibrating in vacuum are

\[
T_p = \frac{\pi \rho_p h a^2}{10} u_0^2; \quad V_p = \frac{8\pi E h^3}{9(1 - \nu^2)} a^2 X^2
\]

(5)

Where \( E \) is the Young’s modulus, \( u_0 = (dX/dt)_{\text{max}} \) and \( \nu \) is the poisson’s ratio. Applying the Rayleigh-Ritz method, the first resonant frequency of the diaphragm is obtained as

\[
f_{\text{vac}} = 0.4745 \frac{hc_p}{a^2}; \quad c_p = \sqrt{\frac{E_p}{(1 - \nu^2) \rho_p}}
\]

(6)

Where \( c_p \) is the velocity of propagation of waves in plate.

2.2 Plate in contact with an inviscid fluid (Acoustic radiation damping)

For a plate in contact with an inviscid and incompressible fluid with the density \( \rho_f \) on one side, the presence of the fluid has the effect of lowering the frequency on account of the increased inertia, and damping of the vibrations owing to the energy carried off in the form of sound-waves. Therefore, this type of damping is known as acoustic radiation or added mass effect. The kinetic energy of the fluid in contact with plate was expressed by Lamb [12] as follows

\[
T_f = 0.2102 \rho_f a^3 u_0^2
\]

(7)

The effect of fluid is virtually to increase the inertia in the ratio of \( \beta \). Where, \( \beta \) is known as the added virtual mass factor.

\[
\frac{T_p + T_f}{T_p} = 1 + \beta; \quad \beta = 0.6689 \frac{\rho_f a}{\rho_p h}
\]

(8)

Therefore, the resonant frequency of plate in fluid is lowered by a factor of

\[
f_f = \frac{f_{\text{vac}}}{\sqrt{1 + \beta}}
\]

(9)

The rate of damping is estimated by calculating the emitted energy in the form of sound waves into the fluid [12], and it is obtained as

\[
\alpha = \frac{5\pi^2 \rho_f f_f^2 a^2}{9 \rho_p (1 + \beta) hc_f}
\]

(10)

Hence, the \( Q \)-factor of acoustic radiation is

\[
Q_{ar} = \frac{\pi f_f}{\alpha} = 1.20 \frac{\rho_p c_f}{\rho_f c_p} (1 + \beta)^{1.5}
\]

(11)

2.3 Plate in contact with a Newtonian fluid (Acoustic radiation and viscous damping)

It was mentioned earlier that beside acoustic radiation term, viscous damping is also a significant part of energy dissipation in microsystems. The viscous damping can be divided in two parts, damping in free space (also called drag force damping) [13], and squeeze film damping which occurs in narrow gaps [20, 21]. The squeeze film damping aroused when the membrane vibrates in parallel with a wall. The air film between the plate and the wall is squeezed so that some of the air flows in and out of the gap. This
flow dissipates the vibrational energy of the membrane and therefore acts as a damper. Obviously, this damping depends on the gap between the diaphragm and the wall. When the plate is very far away from the wall, the damping force will be reduced to the drag force only. In our case the gap is at least two times of the diaphragm radius and therefore the squeeze film term can be neglected [20].

Viscous damping is exerted on the device because of friction between fluid and surface of the resonator. The fluid flow around the resonator is described by Navier-Stokes equation. The simplified Navier-Stokes equation for incompressible flow with constant viscosity is

$$\rho_f (\nabla \cdot \mathbf{v}) \mathbf{v} + \rho_f \frac{\partial \mathbf{v}}{\partial t} = \mu \Delta \mathbf{v} - \nabla p$$

(12)

Where \( p, \mu, \mathbf{v} \) are the pressure, dynamic viscosity, and velocity. This equation cannot be solved analytically, and some approximations should first introduce. Kozlovsky [22] employed the stream function method [23] to analytically calculate the medium damping. He defined a scalar stream function \( \psi(r,z) \), from which the velocity is derived

$$v_r = -\frac{1}{r} \frac{\partial \psi}{\partial z}, \quad v_z = \frac{1}{r} \frac{\partial \psi}{\partial r}$$

(13)

The advantage of this approach is that the continuity equation of an incompressible fluid is automatically satisfied

$$\frac{1}{r} \frac{\partial}{\partial r} (r v_r) + \frac{\partial}{\partial z} v_z = 0$$

(14)

The vorticity \( (\omega = \nabla \times \mathbf{v}) \) in axisymmetrical flow only has one component in \( \theta \) direction, which is denoted by \( \Omega \). The relation between the vorticity and the stream function is written as

$$-\Omega = \frac{\partial}{\partial r} \left( \frac{1}{r} \frac{\partial \psi}{\partial r} \right) + \frac{1}{r} \frac{\partial^2 \psi}{\partial z^2}$$

(15)

Rewriting the Navier-Stokes equation with the vorticity term

$$\frac{\partial}{\partial t} \omega = - (\mathbf{v} \cdot \nabla) \omega + (\omega \nabla) \mathbf{v} + v \nabla^2 \omega$$

(16)

Where \( v = \mu/\rho \) is the kinetic viscosity. The nonlinear terms in square brackets are negligible compared to the diffusion term in case of small plate velocity \((u_0 \approx a)\). They are also negligible compared to the time–derivative term, if \( u_0 \approx \omega a \). Denoting the plate velocity amplitude as \( u_0 = \omega A \), where \( A \) is the amplitude of the vertical displacement of the plate, and recalling that plate elastic theory requires that \( A \approx a \), the condition is verified. By neglecting those nonlinear terms and solving the new linear equation the vorticity \( \Omega \), and the stream function \( \psi \) are obtained. Subsequently, the kinetic energy of the fluid surrounding the diaphragm obtains as [22]

$$T_f = \rho a^3 u_0^2 \left( 0.06689 \pi + \frac{\pi}{10 \sqrt{2}} \xi + O(\xi^3) \right)$$

(17)

Where \( \xi \) is a nondimensional parameter

$$\xi = \sqrt{\frac{v}{4 \omega a^2}}$$

(18)

With recruiting equation (8), the added virtual mass due to both acoustic radiation and viscosity term is
\[
\beta = 0.6689 \frac{p_f a}{p_p h} \left(1 + 1.057 \xi + O(\xi^3)\right)
\]  

(19)

Kozlovsky [22] also calculated the dissipated energy by viscosity as

\[
U_{vis} = 0.44\pi\xi
\]

(20)

And therefore, the \(Q\)-factor of viscosity term obtains as

\[
Q_{vis} = 2\pi \frac{T_p + T_f}{\xi} = 0.95 \left(\frac{1}{\beta} + 1\right)
\]

(21)

In cases where \(\beta \leq 1\), the energy stored in the fluid is much larger than the energy stored in the plate, therefore the kinetic energy of the plate \(T_p\) can be neglected. This assumption eliminates the term \(1/\beta\) in the \(Q_{vis}\) term. The total \(Q\)-factor \(Q_{tot}\) of the resonator is calculated by help of equation (2) as

\[
\frac{1}{Q_{tot}} = \frac{1}{Q_{ar}} + \frac{1}{Q_{vis}}
\]

(22)

3 Experimental works:

3.1 Fabrication

MEMS technology and the sol-gel method for growth of PZT were combined to fabricate the piezoelectric microdiaphragm sensors. The detailed of the fabrication process is mentioned elsewhere [24]. In summary, the fabrication processes are composed of five main steps. A thin TiO\(_2\)/Pt (15/200 nm) layer was first deposited on the top side of a 4-inch silicon-on-insulator (SOI) wafer <100> as the bottom electrode by sputtering. Afterwards, the Pb(Zr\(_{0.52}\)Ti\(_{0.48}\)) (PZT) solution was prepared and was deposited on the surface by sol-gel method. In the third step, the PZT film was wet etched in the diluted HCl: HF: H\(_2\)O solution with the ratio of 50: 1: 50 at the specific areas to open the access to the bottom electrode. In order to minimize parasitic capacitance induced by the patterned electrode wiring, a Si\(_3\)N\(_4\) layer (~350 nm) was deposited by plasma enhanced chemical vapor deposition (PECVD), and patterned by RIE to serve as an insulating layer. The top electrode (10 nm TiO\(_2\)/200 nm Pt) was sputtered and patterned by using lift-off technique. Finally, the diaphragm was released by deep reactive ion etching (DRIE) from the backside. SEM figures of cross-section of the device are shown in Figure 1.
3.2 Measurement Procedure

The dynamic behavior of the piezoelectric circular microdiaphragm based sensors (PCMS) in different medium was first examined by measuring its resonant frequency in vacuum and then subsequently in air and liquid. An Agilent 4294A impedance analyzer was used to characterize the resonant frequency behavior of the PCMS. In vacuum, the frequency was recorded in different ambient pressures using a vacuum chamber. The pressure in the chamber was monitored using a pressure gauge and is controlled by a flow rate valve. For this measurement, a chip containing an array of several diaphragms was packaged by a Patterned PMMA sheet with spacers between the package and the testing floor to prevent the diaphragms from being sealed. Figure 2 shows the setup of this experiment. Using this setup, resonance measurements were conducted at chamber pressures ranging from 1 atmosphere to 0.05 atm.
In order to study the behavior of the PCMS in ethanol, the fluidic cell was connected through capillary tubes (1 mm inner diameter) and adaptive ports to a syringe pump. The fluidic cell was consisted of two patterned PMMA sheets thermally bonded to each other, and the PCMS glued to the PMMA sheet. This setup which is shown in Figure 3 allows the liquid to continuously stream across the piezoelectric membranes with a constant velocity fixed by the syringe pump.

![Figure 2: The setup for characterization of PCMSs in different ambient pressures.](image)

![Figure 3: The setup for characterization of PCMSs in fluid.](image)

4 Results and Discussions:

4.1 Device Characterization

An example of impedance and phase response of a PCMS is shown in Figure 4. The measured resonance frequency $f_r$ and the anti-resonance frequency $f_a$ from the impedance spectrum are used to determine the effective coupling coefficient of the diaphragm [11]. The coupling factor $k^2$ directly influences efficiency of power emission, bandwidth, and sensitivity of the sensor [25]. For piezoelectric materials, the coupling coefficient $k^2$ can be defined as the ratio of the stored mechanical energy in the piezoelectric material per input electrical energy supplied by an electrical source. For instance, the effective electromechanical coupling coefficient $k^2$ of the fabricated PCMS, shown in Figure 4, was 1.51%. In the experimental part of this work, the $Q$ value is measured by the following relation $f_0/\Delta f$, where $f_0$ is the frequency at which the real part of the impedance reaches its maximum, and $\Delta f$ is the width of the peak at its half height. The $f_0$ and $\Delta f$ are
estimated by fitting the measured phase peak using Lorentz function [6]. For instance, the $Q$-factor of the PCMS shown in Figure 4 was 117.439 at a quite low operating frequency of around 134.937 kHz. This value is comparable with that of a typical FBAR (film bulk acoustic resonators) operating at around GHz [7], which means both of them have similar resolution in detecting frequency shift.

![Figure 4](image.png)

Figure 4. (a) Impedance and phase spectrum of a diaphragm with diameter of 0.8 mm shows its resonant frequency is 134.937 kHz. (b) The $Q$-value as high as 117.439 was calculated by fitting the Lorentz function with phase response.

4.2 Resonant frequency behavior

We tested the frequency response of the PCMS at different ambient pressure from normal atmosphere (1 atm = 760 torr) to 0.05 atm. The results of these measurements are shown in Figure 5. As it was expected the resonant frequency of the diaphragm decreases from vacuum to normal atmosphere due to added mass effect. We observed a frequency change around 4.7% from air to 0.05 atm pressure. In lower pressures the peaks also demonstrates nonlinear behavior in their vibration. This nonlinearity which was fully explained in our previous paper [26], originally comes from a change in the restoring force, such as the flexural rigidity or membrane tension, due to large vibration amplitude [27]. In vacuum due to elimination of the added mass effect the vibration amplitude increases and therefore, the nonlinearity in vibration of the PCMS was observed.
We employed equation (9) to calculate the frequency shift of the sensor in each different pressure. In these calculations, the material properties of different layers and the air properties listed in Table 1 and Table 2 were used. The average density of the diaphragm and the total thickness of the plate are \( \rho_p = \sum \rho_i h_i / h_p = 5548 \text{ kg/m}^3 \) and \( h_p = 3.65 \mu\text{m} \), respectively. The density of air is pressure dependent and it was calculated by

\[
\rho_f = 1.18 \times 10^{-5} \frac{p}{p_0}
\]

(23)

Where \( p_0 = 1\text{pa} \) and \( p \) is the ambient pressure. The value of velocity of sound in air which is independent of pressure is shown in Table 1. The comparison between the theoretical and experimental values of resonant frequency in different pressures is summarized in Table 3. We choose the frequency of the diaphragm at 0.05 atm as the resonant frequency in vacuum \( (f_{vac}=58.045 \text{ kHz}) \), and the subsequent values of frequencies calculated based on this frequency. Table 3 clearly demonstrates a good agreement between the proposed theoretical values by Lamb method and the experiment.
We theoretically investigated the frequency behavior of a PCMS in different mediums for a radius range (300µm≤r≤325µm), and the results are summarized in Figure 6. For radius=300 µm, the diaphragm frequency decreases by almost 0.58% from vacuum to air. For that diaphragm vibrates in ethanol, Lamb (Inviscid fluid) and Kozlovsky (Viscous fluid) models estimate the frequency shift of around 66.32% and 66.63% from the vacuum condition. The small difference in the values predicted by these two models is due to low kinetic viscosity of the ethanol (v=1.2×10⁻⁶ m²/s). For higher viscosities, for instance, 10 or 100 times of the ethanol viscosity this frequency shift is 67.27% and 69.07%, respectively. (For having a sense of viscosity values, the blood viscosity is v=3-4×10⁻⁶ m²/s, and the olive oil viscosity is v=9×10⁻⁵ m²/s.)
In order to compare the ratio of contribution of the viscosity over the acoustic radiation terms on frequency shift of a PCMS, Figure 7 was drawn. In this figure the ratio of viscous term over acoustic radiation term ($\beta_{vis}/\beta_{ar} = 1.057\xi$) is plotted over kinematic viscosity. The vertical axis in this figure as stated earlier is proportional to nondimensional parameter $\xi$.

$$\xi = \frac{u}{\omega \alpha^2}; \quad \omega \propto \frac{hc_p}{a^2}$$

By replacing the natural frequency $\omega$ in $\xi$, it was found that, the viscosity effect is mainly dependent to $(u/hc_p)^{0.5}$. This means, besides the kinematic viscosity, the thickness and the sound velocity in the plate are also two significant parameters in viscosity influences on the frequency shift. For instance, for constant sound velocity and viscosity, the influence of viscous term increases by decreasing the thickness $h$. This is the trend in microfabrication technology to reduce the sizes, which shows that the viscosity influences gain higher importance in lower sizes. For our fabricated PCMS, the velocity of sound in the diaphragm was $c_p = 5277 \text{ m/s}$, which was calculated by material properties listed in Table 1. Figure 7 demonstrates that the viscosity influences reach 4.8% in the viscosity of hundred times of ethanol; however, in ethanol range viscosity this contribution can be neglected. This is in agreement with the experimental observations of
Ayela and Nicu [28] that only high viscosities (higher than 10 cp) have a significance influence on frequency shift of the piezoelectric sensor.

![Graph showing relative contribution of viscosity to acoustic radiation term on frequency as a function of kinematic viscosity.](image)

Figure 7: Relative contribution of viscosity to acoustic radiation term on frequency as a function of kinematic viscosity.

### 4.3 Q-factor analysis

The discussion focused so far was on frequency analysis of a PCMS as a function of surrounding fluid. However, as the effect of the added mass and liquid viscosity on the behavior of the PCMS was our primary interest in this study, it would be worthy of studying these parameters on the Q-factor values. As it was mentioned earlier, the Q-factor in the experimental part is defined as $Q = f_0 / \Delta f$, where the values of $f_0$ and $\Delta f$ are obtained by fitting the measured phase peak using Lorentz function. Figure 8 demonstrates the calculated Q values of the frequency peaks shown in Figure 5. As it was expected in lower pressures the air damping reduces and therefore the Q-factor increases. Experimental results show that the Q-factor of the PCMS in 0.05atm is around seven times higher than the time it works in normal atmosphere. We also theoretically calculate the Q-factor of the device in air using equations (11) and (21). The calculation demonstrates that for a diaphragm with radius $a=500 \, \mu m$, $Q_{ar}$ and $Q_{vis}$ are 377 and 2979, respectively. This result clarifies that the viscosity of the air doesn’t play an important role in the $Q_{tot}$ of the diaphragm which vibrates in the air.
Figure 8: The $Q$-factor of PCMS with radius 500 $\mu$m in different pressure from 1 to 0.05 atm.

The theoretical $Q_{th}$ of the PCMS in air for a radius range of $300\mu m \leq r \leq 700\mu m$ is illustrated in Figure 9. This curve is an upper bound of the device $Q$-factor in air, because other damping sources, such as intrinsic or support damping, were neglected in the theoretical section. The $Q$-value for this radius range was between 324 and 346. The highest measured experimental value of $Q$-factor was 137. The $Q$-factors of nine different samples are depicted in Figure 9. The $Q$-factors vary from device to device, even for the same radius. The reason is that the $Q$-factor is dependent to the physical and chemical quality of piezoelectric layer, the thickness of layers, and stress issues in different layers, which may vary case by case. The measured $Q$-values define a rectangular which confines the $Q$-values between the highest and the lowest $Q$-factor measured experimentally.
The $Q$-factor of a PCMS was also investigated in ethanol. Figure 10 shows the phase response of a PCMS in air and in ethanol. The $Q$-factor of that device in air was 111.195, and this amount reduces to 23.908 in ethanol. This is 4.687 times reduction in $Q$-value. It should be outlined that this $Q$-factor is higher than the $Q$-factors of most of state-of-the-art micromachined cantilevers used for specific applications in liquid media. The higher $Q$ value in liquid is main advantage of microdiaphragms over microcantilevers [29]. The theoretical calculation of the $Q$-factor in ethanol for a diaphragm with $a=400\mu m$ results $Q_{ar}=70.64$, $Q_{vi}=123.53$, and therefore $Q_{tot}=44.94$. It is worth emphasizing that the theoretical $Q$-factor calculation in this work is just an estimation of the total $Q$ of the device. Therefore, the high difference in $Q$-values is reasonable.
Figure 11 demonstrates the contribution of viscosity and acoustic radiation on the $Q$-factor of a PCMS with radius $300\mu m \leq r \leq 700\mu m$ working in ethanol. For this radius range the added virtual mass factor changes in the range of $7.88 \leq \beta \leq 18.43$. This value is between 0.012 and 0.028 for a diaphragm working in air. Figure demonstrates that the predominant parameter in $Q_{tot}$ is the acoustic radiation term. $Q_{ar}$ is between 47.90 and 154.42, while the $Q_{vis}$ is between 135.64 and 104.36. The $Q_{tot}$ value is between 35.40 and 62.27. The Figure also shows that the $Q_{medium}$ increases in larger radiuses, which implies that the medium influences have less significance in $Q_{tot}$ of the device in bigger sizes. This in accordance with experimental observations that in macro scale region the intrinsic dissipation mechanisms are the predominant factor in calculation of $Q$-factor of the resonating device.
5 Conclusion

Piezoelectric circular microdiaphragm sensors were fabricated by combining sol-gel PZT thin film and MEMS technology. Their dynamics behavior under medium damping was fully investigated. The contribution of viscosity over acoustic radiation damping is inversely related to the thickness of the diaphragm. Hence, the viscosity has more significance in thinner diaphragms. It was shown that the viscosity influences on frequency reach 4.8% in the viscosity of hundred times of ethanol; however, in ethanol range viscosity this contribution can be neglected. The added virtual mass factor ($\beta$) varies from 0.028 in air to 18.43 in ethanol for a PCMS with radius $a=700\mu m$, which clearly shows significant effect of medium on frequency shift of the microdiaphragm. High $Q$-factor as high as 137 was measured in air. Theoretical calculations of $Q$ for a PCMS with radius $a=400\mu m$ in ethanol results $Q_{ar}=70.64$, $Q_{vis}=123.53$, and therefore $Q_{tot}=44.94$. These values are higher than the $Q$-factor of most of state-of-the-art micromachined cantilevers. This high $Q$ value in liquid is the main advantage of microdiaphragms over microcantilevers.
References


Figure 4 (Fig 4.jpg)
Figure 6 (Fig 6.jpg)
Figure 9 (Fig 9.jpg)
Figure 10 (Fig 10.jpg)
Figure 11 (Fig 11.jpg)

\[ Q_{ar} = 1.20 \frac{\rho_p c_f}{\rho_f c_p} (1 + \beta)^{1.5} \]

\[ Q_{vis} = \frac{0.95}{\xi} \left( \frac{1}{\beta} + 1 \right) \]

\[ \frac{1}{Q_{tot}} = \frac{1}{Q_{ar}} + \frac{1}{Q_{vis}} \]